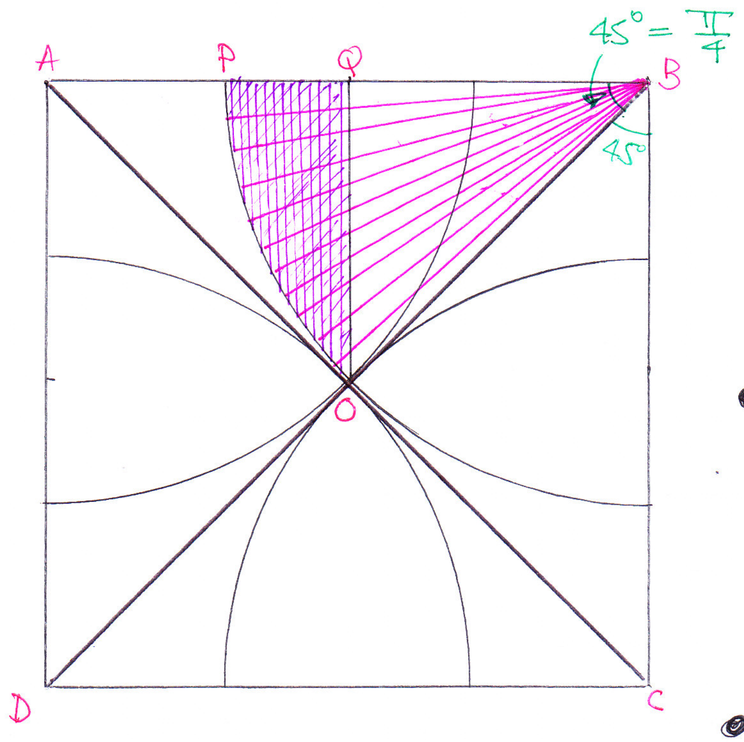
C2, IYGB, PAPER S

1.



- SHAPE IS SYMMETRICAL
- NEED TO FIND THE AREA BOUNDED BY PQ, QO AND THE ARC OP, THEN MULTIPLY BY 8
- AREA OF SECTOR IN "PINK"

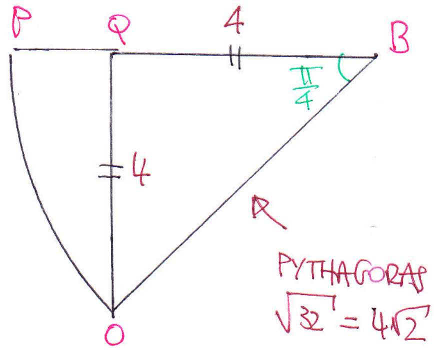
$$\frac{1}{2}r^2\theta^c = \frac{1}{2}(\sqrt{32})^2\frac{\pi}{4}$$

$$= \frac{1}{2}\times 32\times\frac{\pi}{4}$$

$$= 4\pi$$

- AREA OF TRIANGLE IS

$$\frac{1}{2}\times 4\times 4 = 8$$



- "PURPLE AREA" IS

$$4\pi - 8$$

- REQUIRED AREA IS

$$8\times(4\pi-8)$$

$$= 32(\pi-2)$$

AS REQUIRED

2. STARTING FROM THE RHS AND NOTING $\binom{N}{R} = \frac{N!}{R!(N-R)!}$

$$\text{RHS} = \binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(k-1)![(n-1)-(k-1)]!} + \frac{(n-1)!}{k[(n-1)-k]!}$$

$$= (n-1)!\left[\frac{1}{(k-1)!(n-k)!} + \frac{1}{k!(n-k-1)!}\right] = \ldots \text{ ADD FRACTIONS} \ldots$$

$$= (n-1)!\left[\frac{k}{k(k-1)!(n-k)!} + \frac{n-k}{k!(n-k)(n-k-1)!}\right]$$

$$= (n-1)!\left[\frac{k}{k!(n-k)!} + \frac{n-k}{k!(n-k)!}\right] = (n-1)!\left[\frac{n}{k!(n-k)!}\right]$$

$$= \frac{n(n-1)!}{k!(n-k)!} = \frac{n!}{k!(n-k)!} = \binom{n}{k} = \text{LHS}$$

AS REQUIRED

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3. $\frac{\tan x}{\cos x} + \frac{1}{1+\sin x} = \frac{4}{3}$

MULTIPLY THE EQUATION THROUGH BY $3\cos x(1+\sin x)$

$\Rightarrow 3\tan x(1+\sin x) + 3\cos x = 4\cos x(1+\sin x)$

$\Rightarrow \frac{3\sin x}{\cos x}(1+\sin x) + 3\cos x = 4\cos x(1+\sin x)$

$\Rightarrow 3\sin x(1+\sin x) + 3\cos^2 x = 4\cos^2 x(1+\sin x)$

$\Rightarrow 3\sin x + 3\sin^2 x + 3\cos^2 x = 4(1-\sin^2 x)(1+\sin x)$

$\Rightarrow 3\sin x + 3(\sin^2 x+\cos^2 x) = 4(1-\sin^2 x)(1+\sin x)$

$\Rightarrow 3\sin x + 3 = 4(1-\sin^2 x)(1+\sin x)$

$\Rightarrow 3(\sin x+1) = 4(\sin x+1)(1-\sin^2 x)$

$\sin x = -1$ IS A SOLUTION BY INSPECTION, BUT $\sin x \neq -1$ BECAUSE THEN $\cos x = 0$, SO WE MAY CORRECTLY DIVIDE IT OUT

$\Rightarrow 3 = 4(1-\sin^2 x)$

$\Rightarrow \frac{3}{4} = 1 - \sin^2 x$

$\Rightarrow \sin^2 x = \frac{1}{4}$

$\Rightarrow \sin x = \pm\frac{1}{2}$

- $\arcsin\left(\frac{1}{2}\right) = 30°$

$\begin{cases} x = 30 \pm 360n \\ x = 150 \pm 360n \end{cases}$

$n = 0, 1, 2, 3, \ldots$

- $\arcsin\left(-\frac{1}{2}\right) = -30$

$\begin{cases} x = -30 \pm 360n \\ x = 210 \pm 360n \end{cases}$

$n = 0, 1, 2, 3, \ldots$

$x = 30°, 150°, 330°, 210°$

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Q3 VARIATION

$$\frac{\tan x}{\cos x} + \frac{1}{1+\sin x} = \frac{4}{3}$$

$$\frac{\frac{\sin x}{\cos x}}{\cos x} + \frac{1}{1+\sin x} = \frac{4}{3}$$

$$\frac{\sin x}{\cos^2 x} + \frac{1}{1+\sin x} = \frac{4}{3}$$

$$\frac{\sin x(1+\sin x) + \cos^2 x}{\cos^2 x(1+\sin x)} = \frac{4}{3}$$

$$\frac{\sin x + \sin^2 x + \cos^2 x}{\cos^2 x(1+\sin x)} = \frac{4}{3}$$

$$\frac{\sin x + 1}{\cos^2 x(1+\sin x)} = \frac{4}{3}$$

CANCEL $1+\sin x$ DUE TO THE EXPLANATION GIVEN EARLIER

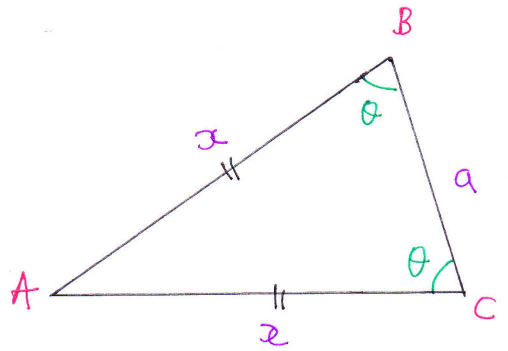
$$\frac{1}{\cos^2 x} = \frac{4}{3}$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm\frac{\sqrt{3}}{2}$$

ETC

4.



- BY THE COSINE RULE

$$|AC|^2 = |AB|^2 + |BC|^2 - 2|AB||BC|\cos\theta$$

$$\cancel{x^2} = \cancel{x^2} + a^2 - 2xa\cos\theta$$

$$2ax\cos\theta = a^2$$

$$2x\cos\theta = a \qquad (a \neq 0)$$

$$\boxed{x = \frac{a}{2\cos\theta}}$$

HENCE THE AREA IS GIVEN BY $= \frac{1}{2}|AB||BC|\sin\theta$

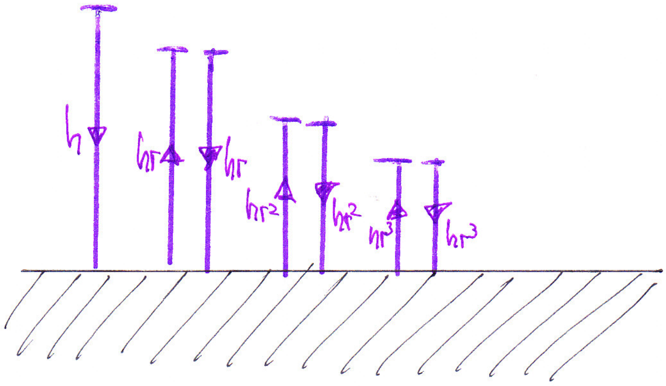
$$= \frac{1}{2} x a \sin\theta$$

$$= \frac{1}{2} \times \frac{a}{2\cos\theta} \times a \times \sin\theta$$

$$= \frac{1}{4}a^2\tan\theta$$

AS REQUIRED

5.



$\Rightarrow d = h + 2hr + 2hr^2 + 2hr^3 + \dots$

$\Rightarrow d = h[1 + 2r + 2r^2 + 2r^3 + \dots]$

$\Rightarrow d = h[1 + 2\underbrace{(r + r^2 + r^3 + \dots)}_{\text{This is a G.P with } a = r,\ r = r}]$

This is a G.P with $a = r$
$r = r$

$S_\infty = \frac{a}{1-r}$

HERE

$S_\infty = \frac{r}{1-r}$

$\Rightarrow d = h\left[1 + 2\left(\frac{r}{1-r}\right)\right]$

$\Rightarrow d = h\left[1 + \frac{2r}{1-r}\right]$

$\Rightarrow d = h\left(\frac{1-r+2r}{1-r}\right)$

$\Rightarrow d = h\left(\frac{1+r}{1-r}\right)$

$\Rightarrow d(1-r) = h(1+r)$

$\Rightarrow d - dr = h + hr$

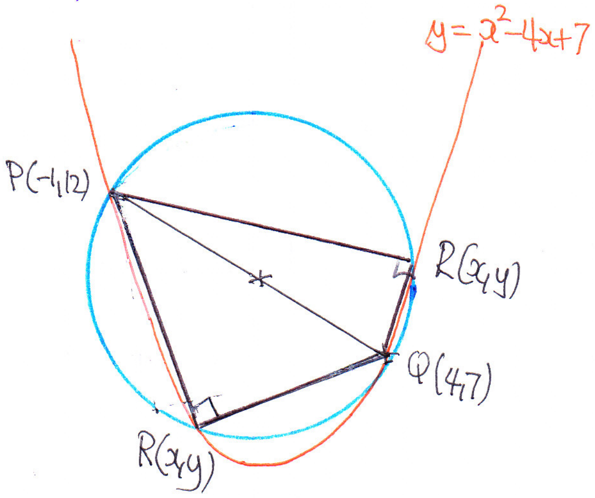
$\Rightarrow d - h = hr + dr$

$\Rightarrow d - h = r(h+d)$

$\Rightarrow r = \frac{d-h}{d+h}$

AS REQUIRED

6. THE POINTS P, Q & R MUST LIE ON A CIRCLE, AS SHOWN IN THE DIAGRAM OPPOSITE.



- OBTAIN THE EQUATION OF THE CIRCLE BY GRADIENTS, GIVEN PQ IS A DIAMETER
- GRAD PR $= \frac{y-12}{x+1}$
- GRAD QR $= \frac{y-7}{x-4}$
- $\frac{y-12}{x+1} \times \frac{y-7}{x-4} = -1$

$$\frac{y^2 - 19y + 84}{x^2 - 3x - 4} = -1$$

$$y^2 - 19y + 84 = -x^2 + 3x + 4$$

$$\boxed{y^2 + x^2 - 3x - 19y + 80 = 0}$$

$(a+b+c)^2 \equiv a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

SOLVING SIMULTANEOUSLY WITH $y = x^2 - 4x + 7$

$\Rightarrow (x^2 - 4x + 7)^2 + x^2 - 3x - 19(x^2 - 4x + 7) + 80 = 0$

$\Rightarrow x^4 + 16x^2 + 49 - 8x^3 + 14x^2 - 56x + x^2 - 3x - 19x^2 + 76x - 133 + 80 = 0$

$\Rightarrow x^4 - 8x^3 + 12x^2 + 17x - 4 = 0$

NOW $x = -1$ (POINT P) & $x = 4$ (POINT Q) ARE SOLUTIONS SO WE MAY DIVIDE THEM OUT

$\Rightarrow x^3(x+1) - 9x^2(x+1) + 21x(x+1) - 4(x+1) = 0$

$\Rightarrow (x+1)(x^3 - 9x^2 + 21x - 4) = 0$

$\Rightarrow (x+1)\left[x^2(x-4) - 5x(x-4) + (x-4)\right] = 0$

$\Rightarrow \underbrace{(x+1)}_{P}\ \underbrace{(x-4)}_{Q}\ \underbrace{(x^2 - 5x + 1)}_{R} = 0$

THUS $x^2-5x+1=0 \implies x = \dfrac{5 \pm \sqrt{25-4\times1\times1}}{2\times1}$

$$\implies \boxed{x = \frac{5\pm\sqrt{21}}{2}}$$

$y = x^2-4x+7$

$$\implies y = \left(\frac{5\pm\sqrt{21}}{2}\right)^2 - 4\left(\frac{5\pm\sqrt{21}}{2}\right)+7$$

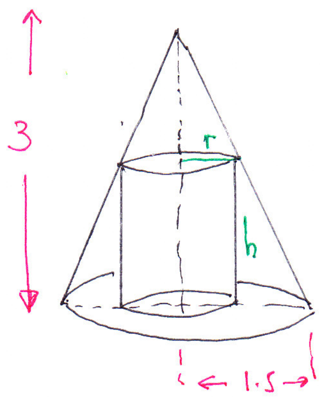
$$\implies y = \frac{25+21\pm10\sqrt{21}}{4} - 10 \mp 2\sqrt{21}+7$$

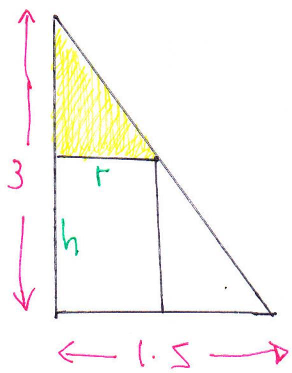
$$\implies y = \frac{23}{2} \pm \frac{5}{2}\sqrt{21} - 3 \mp 2\sqrt{21}$$

$$\implies y = \frac{17}{2} \begin{cases} +\frac{1}{2}\sqrt{21} \\ -\frac{1}{2}\sqrt{21} \end{cases}$$

$$\therefore\ R\left(\frac{5+\sqrt{21}}{2}, \frac{17+\sqrt{21}}{2}\right) \text{ OR } R\left(\frac{5-\sqrt{21}}{2}, \frac{17-\sqrt{21}}{2}\right)$$

7.





- LET r & h BE THE RADIUS & HEIGHT OF THE CYLINDER
- BY SIMILAR TRIANGLES LOOKING AT THE YELLOW TRIANGLE & THE ENTIRE TRIANGLE

$$\frac{r}{3-h} = \frac{1.5}{3}$$

$$\frac{r}{3-h} = \frac{1}{2}$$

$$2r = 3-h$$

$$\boxed{h = 3-2r}$$

P.T.O

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- VOLUME OF A CYLINDER

$$V = \pi r^2 h$$

$$V = \pi r^2(3-2r)$$

$$V = \pi(3r^2 - 2r^3)$$

- BY DIFFERENTIATION

$$\frac{dV}{dr} = \pi(6r - 6r^2)$$

$$\frac{d^2V}{dr^2} = \pi(6 - 12r)$$

- SOLVE $\frac{dV}{dr} = 0$

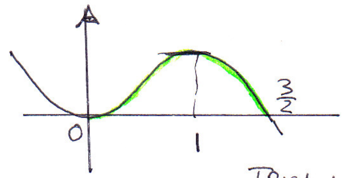
$$6\pi r(1-r) = 0$$

$$r \neq 0$$

$$\therefore \boxed{r=1}$$

- $\left.\frac{d^2V}{dr^2}\right|_{r=1} = -6\pi < 0$ IE LOCAL MAX

SO



TRUE MAX

- WHEN $r=1$

$$V = \pi(3\times1^2 - 2\times1^3)$$

$$V = \pi$$

AS REQUIRED

8.

$$\frac{2-\log_4 x^7}{7-\log_4 x^2} + (\log_4 x)^2 = 0$$

$$\Rightarrow \frac{2 - 7\log_4 x}{7 - 2\log_4 x} = -(\log_4 x)^2$$

$$\Rightarrow \frac{2-7y}{7-2y} = -y^2$$

WHERE $y = \log_4 x$

$$\Rightarrow 2-7y = -y^2(7-2y)$$

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$\Rightarrow 7y - 2 = y^2(7 - 2y)$

$\Rightarrow 7y - 2 = 7y^2 - 2y^3$

$\Rightarrow 2y^3 - 7y^2 + 7y - 2 = 0$

LOOK FOR A FACTOR BY INSPECTION HERE (EASY LOOKING AT THESE COEFFICIENTS)

$\therefore$ $y = 1$ IS A SOLUTION

BY LONG DIVISION, INSPECTION, OR MANIPULATION

$\Rightarrow 2y^2(y-1) - 5y(y-1) + 2(y-1) = 0$

$\Rightarrow (y-1)(2y^2 - 5y + 2) = 0$

$\Rightarrow (y-1)(2y-1)(y-2) = 0$

$\Rightarrow y = \begin{cases} 1 \\ \frac{1}{2} \\ 2 \end{cases}$

$\Rightarrow \log_4 x = \begin{cases} 1 \log_4 4 = \log_4 4 \\ \frac{1}{2} \log_4 4 = \log_4 2 \\ 2 \log_4 4 = \log_4 16 \end{cases}$

$\Rightarrow x = \begin{cases} 4 \\ 2 \\ 16 \end{cases}$

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9.

$y = 1 + 2x - x^2$

$\frac{dy}{dx} = 2 - 2x$

$\left.\frac{dy}{dx}\right|_A = \left.\frac{dy}{dx}\right|_{x=0} = 2$

$\therefore$ NORMAL GRADIENT $= -\frac{1}{2}$

EQUATION OF NORMAL

$\boxed{y = -\frac{1}{2}x + 1}$

SOLVING SIMULTANEOUSLY TO FIND THE CO-ORDINATES OF B

$\left.\begin{aligned} y &= -\tfrac{1}{2}x + 1 \\ y &= 1 + 2x - x^2 \end{aligned}\right\} \Rightarrow$

$\Rightarrow -\frac{1}{2}x + \cancel{1} = \cancel{1} + 2x - x^2$

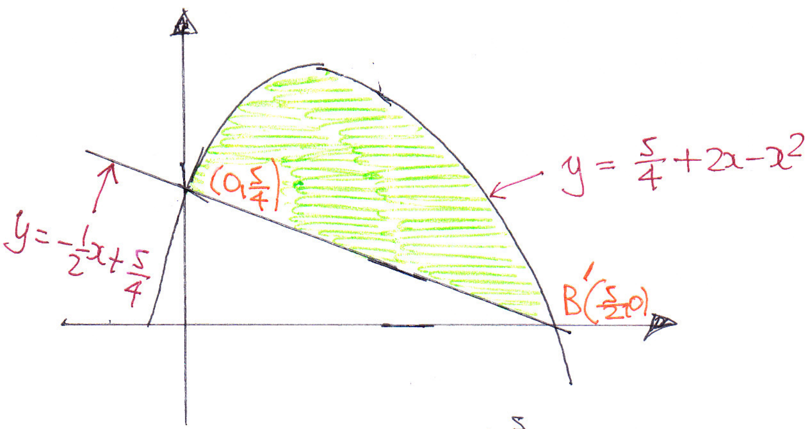
$\Rightarrow x^2 - \frac{5}{2}x$

$\Rightarrow \frac{1}{2}x(2x - 5)$

$x = \begin{cases} 0 \\ \frac{5}{2} \end{cases} \quad y = \begin{cases} 1 \\ -\frac{1}{4} \end{cases}$

$\therefore B\left(\frac{5}{2}, -\frac{1}{4}\right)$

NOW METHOD A — TRANSLATE "THE PICTURE" UP BY $\frac{1}{4}$



AREA UNDER THE CURVE $= \int_0^{\frac{5}{2}} \frac{5}{4} + 2x - x^2 \, dx$

$= \left[\frac{5}{4}x + x^2 - \frac{1}{3}x^3\right]_0^{\frac{5}{2}}$

$= \left(\frac{25}{8} + \frac{25}{4} - \frac{125}{24}\right) - 0$

$= \frac{75 + 150 - 125}{24} = \frac{100}{24} = \frac{25}{6}$

AREA OF TRIANGLE $= \frac{1}{2} \times \frac{5}{2} \times \frac{5}{4} = \frac{25}{16}$

$\therefore$ REQUIRED AREA $= \frac{25}{6} - \frac{25}{16} = 25\left(\frac{1}{6} - \frac{1}{16}\right) = 25\left(\frac{8}{48} - \frac{3}{48}\right)$

$= 25 \times \frac{5}{48} = \frac{125}{48}$

METHOD B INTEGRATE BETWEEN THE OBJECTS, BETWEEN $x=0$ & $x=\frac{5}{2}$

$$\text{AREA} = \int_0^{5/2} (\cancel{1}+2x-x^2) - (-\tfrac{1}{2}x\cancel{+1})\, dx$$

$$= \int_0^{5/2} \tfrac{5}{2}x - x^2\, dx$$

$$= \left[\tfrac{5}{4}x^2 - \tfrac{1}{3}x^3\right]_0^{5/2}$$

$$= \left(\tfrac{125}{16} - \tfrac{125}{3}\right) - (0)$$

$$= 125\left(\tfrac{1}{16} - \tfrac{1}{24}\right)$$

$$= 125\left(\tfrac{3}{48} - \tfrac{2}{48}\right)$$

$$= \tfrac{125}{48}$$

AS BEFORE.