C2, IYGB, PAPER L

1. $(2-5x)^5 = \binom{5}{0}(2)^5(-5x)^0 + \binom{5}{1}(2)^4(-5x)^1 + \binom{5}{2}(2)^3(-5x)^2 + \dots$

$= (1\times 32\times 1) + (5\times 16\times(-5x)) + (10\times 8\times 25x^2) + \dots$

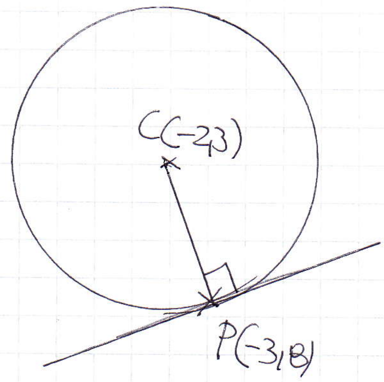
$= 32 - 400x + 2000x^2 + \dots$

2. a) $\text{RADIUS} = \sqrt{(-3+2)^2+(8-3)^2}$

$= \sqrt{1+25}$

$= \sqrt{26}$

$(x+2)^2 + (y-3)^2 = 26$



b) $\text{GRAD C.P} = \frac{y_2-y_1}{x_2-x_1} = \frac{8-3}{-3+2} = \frac{5}{-1} = -5$

TANGENT GRADIENT MUST BE $\frac{1}{5}$

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 8 = \frac{1}{5}(x+3)$

$\Rightarrow 5y - 40 = x + 3$

$\Rightarrow 0 = x - 5y + 43$ $\quad$ ie $x - 5y + 43 = 0$

AS REQUIRED

3. $\log_5(4t+7) - \log_5 t = 2$

$\Rightarrow \log_5\left(\frac{4t+7}{t}\right) = 2\log_5 5$

$\Rightarrow \log_5\left(\frac{4t+7}{t}\right) = \log_5 25$

$\Rightarrow \frac{4t+7}{t} = 25$

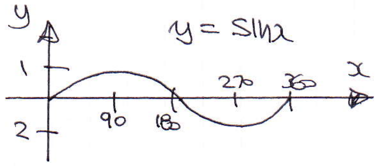
$\Rightarrow 4t+7 = 25t$

$\Rightarrow 7 = 21t$

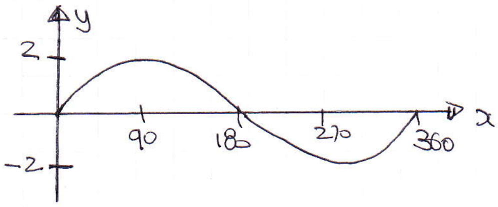
$\Rightarrow t = \frac{7}{21}$ $\quad \therefore t = \frac{1}{3}$

C2, IYGB, PAPER L

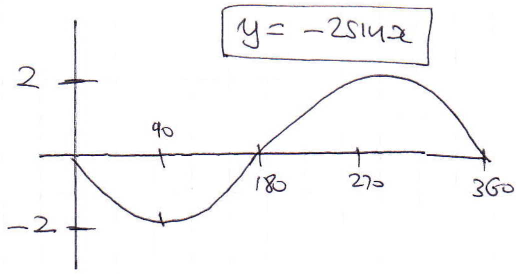
4. a) THE GRAPH OF $\sin x$ IS SHOWN OPPOSITE



• NOW $\boxed{y = 2\sin x}$



• $\boxed{y = -2\sin x}$



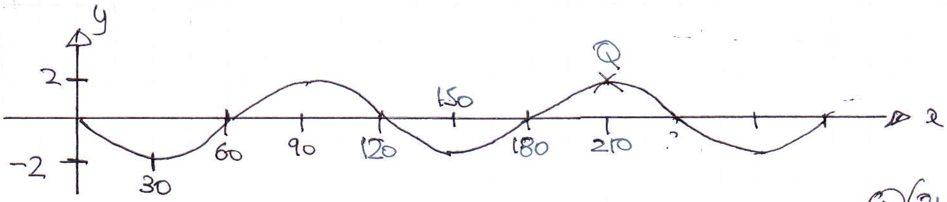
So $A = -2$

$90 \div 3$

AND SINCE THE FIRST MIN OCCURS AT $x = 30$, THIS MUST BE $y = -2\sin 3x$

$\therefore k = 3$

b) THUS



$Q(210, 2)$

5. • $a + ar + ar^2 + ar^3 = 1800$ • $\cancel{a}r^3 = 8\cancel{a}$

$r^3 = 8$

$\boxed{r = 2}$

$\Longrightarrow \boxed{a(1 + r + r^2 + r^3) = 1800}$

$\Longrightarrow a(1 + 2 + 4 + 8) = 1800$

$\Longrightarrow 15a = 1800$

$\Longrightarrow \boxed{a = 120}$

$\therefore$ $a = 120$

$ar = 240$

$ar^2 = 480$

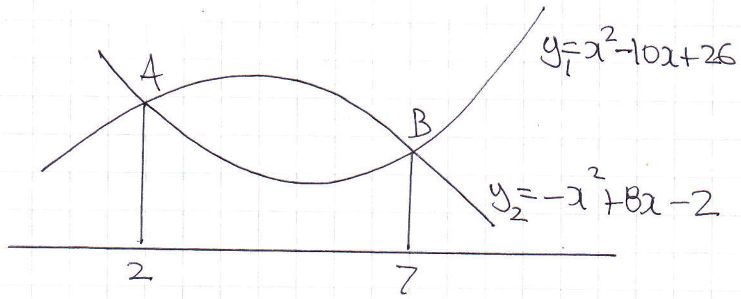
$ar^3 = 960$

ie £120, £240, £480, £960

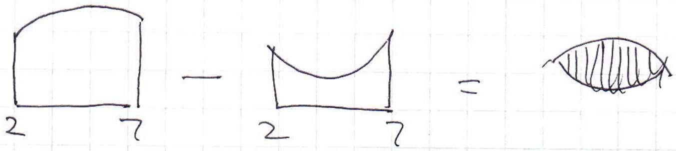
6. FIRSTLY FIND THE CO-ORDINATES OF A & B

$$-x^2+8x-2 = x^2-10x+26$$
$$0 = 2x^2-18x+28$$
$$0 = x^2-9x+14$$
$$0 = (x-7)(x-2)$$
$$x = \begin{cases} 2 \leftarrow A \\ 7 \leftarrow B \end{cases}$$

(y co-ords not needed)



THUS



$$\int_2^7 -x^2+8x-2 \, dx$$
$$= \left[-\tfrac{1}{3}x^3+4x^2-2x\right]_2^7$$
$$= \left(-\frac{343}{3}+196-14\right)-\left(-\frac{8}{3}+16-4\right)$$
$$= \frac{203}{3}-\left(\frac{28}{3}\right)$$
$$= \frac{175}{3}$$

$$\int_2^7 x^2-10x+26 \, dx$$
$$= \left[\tfrac{1}{3}x^3-5x^2+26x\right]_2^7$$
$$= \left(\frac{343}{3}-245+182\right)-\left(\frac{8}{3}-20+52\right)$$
$$= \frac{154}{3}-\frac{104}{3}$$
$$= \frac{50}{3}$$

REQUIRED AREA $= \frac{175}{3}-\frac{50}{3} = \frac{125}{3}$

AS REQUIRED

ALTERNATIVE METHOD

$$\int_2^7 (y_2-y_1) \, dx$$
$$= \int_2^7 -2x^2+18x-28 \, dx$$
$$= \left[-\tfrac{2}{3}x^3+9x^2-28x\right]_2^7$$
$$= \left(\frac{-683}{3}+441-196\right)-\left(-\frac{16}{3}+36-56\right)$$
$$= \frac{49}{3}-\left(-\frac{76}{3}\right) = \frac{125}{3}$$

C2, IYGB, PAPER L

7. a)

x	0	0.4	0.8	1.2	1.6	2
y	0	1.2	1.6	1.8330	1.9596	2

AREA $\simeq \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times\text{REST}\right]$

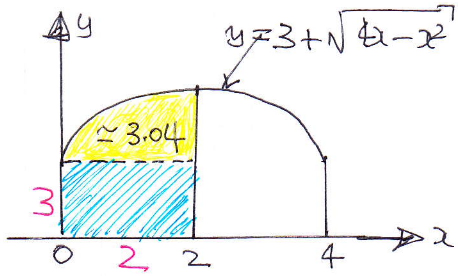
AREA $\simeq \frac{0.4}{2}\left[0+2+2(1.2+1.6+1.8330+1.9596)\right]$

AREA $\simeq 3.03716$

AREA $\simeq 3.04$

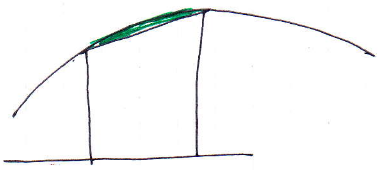
b) $\therefore$ REQUIRED ANSWER IS

$3.04 + (2\times 3) = 9.04$

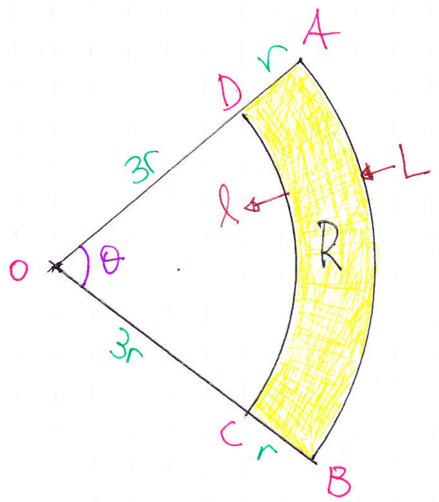


c) THE TRAPEZIUM RULE IN THIS CASE UNDERESTIMATES THE AREA (SEE OPPOSITE DIAGRAM)

SO IF WE TAKE MORE TRAPEZIUMS THE ANSWER WILL INCREASE



8. a)



- AREA OF R $= \frac{1}{2}(4r)^2\theta - \frac{1}{2}(3r)^2\theta$

$50 = 8r^2\theta - \frac{9}{2}r^2\theta$

$100 = 16r^2\theta - 9r^2\theta$

$\boxed{100 = 7r^2\theta}$

$P = l + L + r + r$

$P = 3r\theta + 4r\theta + 2r$

$\boxed{P = 7r\theta + 2r}$

C2, IYGB, PAPER L

SO $\boxed{7r^2\theta = 100}$ & $\boxed{P = 7r\theta + 2r}$

$7r\theta = \frac{100}{r}$

$\therefore\ P = \frac{100}{r} + 2r$ AS REQUIRED

b) I) $P = 2r + 100r^{-1}$

$\frac{dP}{dr} = 2 - 100r^{-2}$

SOLVE FOR ZERO

$2 - \frac{100}{r^2} = 0$

$2 = \frac{100}{r^2}$

$2r^2 = 100$

$r^2 = 50$

$r = +5\sqrt{2}$

II) $\frac{d^2P}{dr^2} = 200r^{-3}$

$\frac{d^2P}{dr^2} = \frac{200}{r^3}$

$\left.\frac{d^2P}{dr^2}\right|_{r=5\sqrt{2}} = \frac{200}{(5\sqrt{2})^3} = \frac{2}{5}\sqrt{2} > 0$

INDEED A MIN

c) $P_{MIN} = 2(5\sqrt{2}) + \frac{100}{5\sqrt{2}} = 10\sqrt{2} + 10\sqrt{2} = 20\sqrt{2}$

9. $(\sqrt{3} + 2\sin 2y)(\sqrt{3} + \tan 2y) = 0$

EITHER

$\sqrt{3} + 2\sin 2y = 0$

$\sin 2y = -\frac{\sqrt{3}}{2}$

$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

$2y = -\frac{\pi}{3} \pm 2n\pi$ $n = 0,1,2,3..$

$2y = \frac{4\pi}{3} \pm 2n\pi$

$y = -\frac{\pi}{6} \pm n\pi$

$y = \frac{2\pi}{3} \pm n\pi$

OR

$\sqrt{3} + \tan 2y = 0$

$\tan 2y = -\sqrt{3}$

$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$

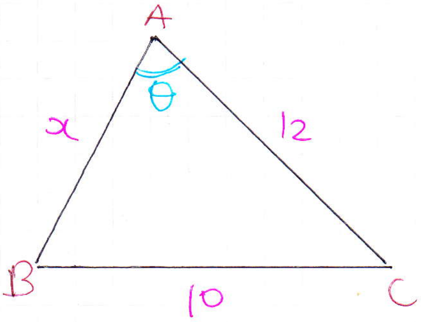
$2y = -\frac{\pi}{3} \pm n\pi$ $n = 0,1,2,3,\ldots$

$y = -\frac{\pi}{6} \pm \frac{n\pi}{2}$

FOR $0 \le y < \pi$

$$y = \frac{5\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{3}$$

10. a)



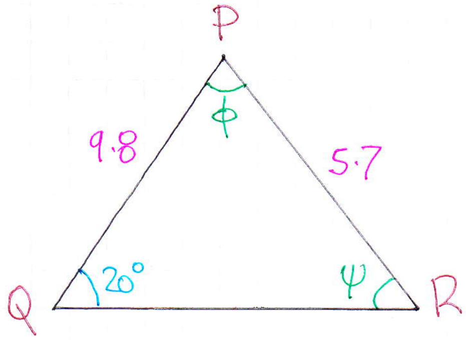
$\boxed{\cos\theta = \frac{5}{9}}$

- BY THE COSINE RULE.

$$|BC|^2 = |AB|^2 + |AC|^2 - 2|AB||AC|\cos\theta$$
$$10^2 = x^2 + 12^2 - 2 \times x \times 12 \times \frac{5}{9}$$
$$100 = x^2 + 144 - \frac{40}{3}x$$
$$300 = 3x^2 + 432 - 40x$$
$$0 = 3x^2 - 40x + 132$$
$$0 = (3x - 22)(x - 6)$$
$$x = \begin{cases} 6 \\ \frac{22}{3} \end{cases}$$

FROM THE FACT "SHORT IS 6m"

b)



- BY THE SINE RULE

$$\frac{5.7}{\sin 20°} = \frac{9.8}{\sin\psi}$$
$$5.7\sin\psi = 9.8\sin 20$$
$$\sin\psi = 0.588...$$
$$\psi \approx \begin{cases} 36.0° \\ 144.0° \end{cases}$$

$$\therefore \phi \simeq \begin{cases} 180 - 20 - 36° \simeq 124° \\ 180 - 20 - 144° \simeq 16° \end{cases}$$