C1, IYGB, PAPER Y

1. a) I) $\dfrac{6}{2^{-2}} = 6\times 2^2 = 6\times 4 = 24$

OR

$\boxed{\dfrac{6}{2^{-2}} = \dfrac{6}{\frac{1}{2^2}} = \left(\dfrac{\frac{6}{1}}{\frac{1}{4}}\right) = 24}$

II) $\left(1\frac{7}{9}\right)^{\frac{3}{2}} = \left(\frac{16}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{16}{9}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

b)

$$z^{\frac{3}{2}} = 27$$
$$\left(z^{\frac{3}{2}}\right)^{\frac{2}{3}} = (27)^{\frac{2}{3}}$$
$$z^1 = \left(\sqrt[3]{27}\right)^2$$
$$z = 9$$

OR

$$z^{\frac{3}{2}} = 27$$
$$\left(\sqrt{z}\right)^3 = 27$$
$$\sqrt{z} = 3$$
$$z = 9$$

2. a) $\dfrac{36}{5-\sqrt{7}} = \dfrac{36(5+\sqrt{7})}{(5-\sqrt{7})(5+\sqrt{7})} = \dfrac{36(5+\sqrt{7})}{25+\cancel{5\sqrt{7}}-\cancel{5\sqrt{7}}-7} = \dfrac{36(5+\sqrt{7})}{18}$

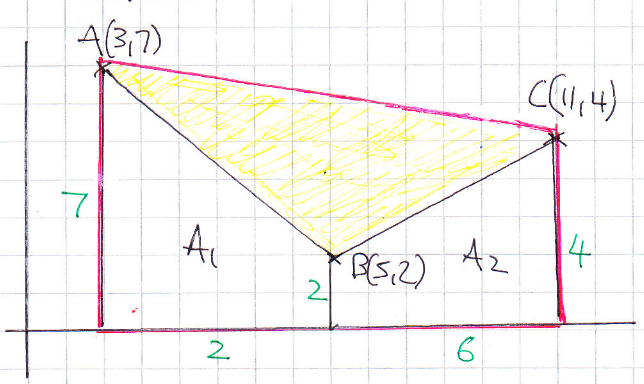
$= 2(5+\sqrt{7}) = 10+2\sqrt{7}$

b) $\sqrt{\dfrac{8}{3}} + \dfrac{3}{2}\sqrt{\dfrac{8}{27}} = \dfrac{\sqrt{8}}{\sqrt{3}} + \dfrac{3}{2}\dfrac{\sqrt{8}}{\sqrt{27}} = \dfrac{2\sqrt{2}}{\sqrt{3}} + \cancel{\dfrac{3}{2}}\times\cancel{\dfrac{2\sqrt{2}}{3\sqrt{3}}}$

$= \dfrac{2\sqrt{2}}{\sqrt{3}} + \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{3\sqrt{2}}{\sqrt{3}} = \dfrac{3\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$= \dfrac{3\sqrt{6}}{3} = \sqrt{6}$

3. METHOD A



$A_1 = \dfrac{7+2}{2}\times 2 = 9$

$A_2 = \dfrac{2+4}{2}\times 6 = 18$

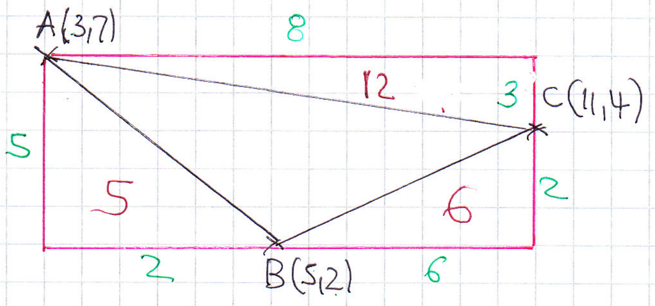
"RED TRAPEZIUM" $= \dfrac{7+4}{2}\times 8 = 44$

REQUIRED AREA $= 44-27$

$= 17$

METHOD B

REQUIRED AREA $= (5\times8) - 12 - 6 - 5 = 40 - 23 = 17$



METHOD C BY DETERMINANTS

$$\text{AREA} = \frac{1}{2}\left\|\begin{matrix}1 & 1 & 1\\ 3 & 5 & 11\\ 7 & 2 & 4\end{matrix}\right\| = \frac{1}{2}\Big[(5\times4)-(2\times11)+(7\times11-3\times4)+(3\times2-7\times5)\Big]$$

$$= \frac{1}{2}\Big[-2+65-29\Big] = \frac{1}{2}\times34 = 17$$

4.

- LET THE GRADIENT OF THE LINE BE m, SO L HAS EQUATION $y = mx - 1$

- $\left.\begin{matrix} y = x^2 + 2x \\ y = mx - 1\end{matrix}\right\} \Rightarrow x^2 + 2x = mx - 1$

$\Rightarrow x^2 + 2x - mx + 1 = 0$

$\Rightarrow x^2 + (2-m)x + 1 = 0$

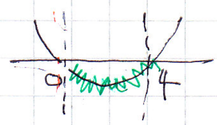
NO INTERSECTIONS $\Rightarrow b^2 - 4ac < 0$

$(2-m)^2 - 4\times1\times1 < 0$

$4 - 4m + m^2 - 4 < 0$

$m^2 - 4m < 0$

$m(m-4) < 0$



$0 < m < 4$

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5.

$$\left.\begin{array}{l} 2x-3y-12=0 \\ x^2+y^2+8y=101 \end{array}\right\} \Longrightarrow \boxed{x=\frac{3}{2}y+6}$$

SUB INTO THE OTHER

$$\left(\frac{3}{2}y+6\right)^2+y^2+8y=101$$

$$\frac{9}{4}y^2+18y+36+y^2+8y=101$$

$$\frac{13}{4}y^2+26y-65=0 \qquad \text{(NOTE 13, 26, 65)}$$

$$13y^2+104y-260=0$$

$$y^2+8y-20=0$$

$$(y+10)(y-2)=0$$

$$y=\begin{cases}2\\-10\end{cases} \qquad x=\begin{cases}9\\-9\end{cases}$$

$$\therefore (9,2) \ \& \ (-9,-10)$$

6.

$$\frac{dy}{dx}=2x-6$$

$$y=\int 2x-6 \, dx$$

$$\boxed{y=x^2-6x+c}$$

$$\left.\begin{array}{l} (a,b) \Rightarrow b=a^2-6a+c \\ (2a,2b) \Rightarrow 2b=4a^2-12a+c \end{array}\right\} \Rightarrow$$

ELIMINATE b

$$2(a^2-6a+c)=4a^2-12a+c$$

$$2a^2-\cancel{12a}+2c=4a^2-\cancel{12a}+c$$

$$c=2a^2$$

$$\therefore y=x^2-6x+2a^2$$

C1, IYGB, PAPER Y

7. a) I)

$$y = \frac{x^2}{2} - \frac{4}{x}$$

$$0 = \frac{x^2}{2} - \frac{4}{x}$$

$$\frac{4}{x} = \frac{x^2}{2}$$

$$x^3 = 8$$

$$x = 2$$

$$\therefore P(2,0)$$

II)

$$y = \frac{1}{2}x^2 - 4x^{-1}$$

$$\frac{dy}{dx} = x + 4x^{-2} = x + \frac{4}{x^2}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = 2 + \frac{4}{2^2} = 2 + 1 = 3$$

NORMAL GRADIENT IS $-\frac{1}{3}$

$$y - y_0 = m(x - x_0)$$

$$y - 0 = -\frac{1}{3}(x - 2)$$

$$3y = -x + 2$$

$$x + 3y = 2$$

b)

$$\left.\begin{aligned} y &= \frac{x^2}{2} - \frac{4}{x} \\ x + 3y &= 2 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 3y &= \frac{3}{2}x^2 - \frac{12}{x} \\ 3y &= 2 - x \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \frac{3}{2}x^2 - \frac{12}{x} = 2 - x$$

$$\Rightarrow 3x^2 - \frac{24}{x} = 4 - 2x$$

$$\Rightarrow 3x^3 - 24 = 4x - 2x^2$$

$$\Rightarrow 3x^3 + 2x^2 - 4x - 24 = 0$$

$$\Rightarrow (x-2)(3x^2 + 8x + 12) = 0$$

POINT OF NORMALITY

BY INSPECTION OR LONG DIVISION

$$b^2 - 4ac = 8^2 - 4 \times 3 \times 12$$

$$= 64 - 144$$

$$= -80 < 0$$

NO SOLUTION

$\therefore$ NORMAL DOES NOT RE INTERSECT

C1, IYGB, PAPER Y

a)

$$\boxed{u_n = a + (n-1)d}$$

$$L = a + (n-1)d$$

$$L - a = (n-1)d$$

$$\frac{L-a}{d} = n-1$$

$$\boxed{n = \frac{L-a}{d} + 1}$$

$$\boxed{S_n = \frac{n}{2}\left[a + L\right]}$$

$$S_n = \frac{1}{2}n\left[a+L\right]$$

$$S_n = \frac{1}{2}\left[L+a\right]\left[\frac{L-a}{d}+1\right]$$

AS REQUIRED

b)

$$550 + 561 + 572 + \cdots + 1100$$

$$\boxed{\begin{aligned} a &= 550 \\ d &= 11 \\ \ell &= 1100 \end{aligned}}$$

$$\therefore S_{UM} = \frac{1}{2}\left[550+1100\right]\left[\frac{1100-550}{11}+1\right]$$

$$= \frac{1}{2} \times 1650 \times 51$$

$$= 825 \times 51$$

$$= 42075$$

$$\begin{array}{r} 825 \\ \times\ 51 \\ \hline 825 \\ 4125 \\ \hline 42075 \end{array}$$

9.

$$\boxed{u_{n+1} = \frac{Au_n + 2}{4 + Bu_n} \qquad u_1 = \frac{1}{2}}$$

$$u_2 = \frac{Au_1 + 2}{4 + Bu_1} \implies \cancel{-2 = \frac{\frac{1}{2}A + 2}{4 + \frac{1}{2}B}}$$

$$u_3 = \frac{Au_2 + 2}{4 + Bu_2} \implies \cancel{-\frac{1}{3} = \frac{-2A+2}{4-2B}}$$

$$\implies$$

$$\left.\begin{aligned} -8 - B &= \tfrac{1}{2}A + 2 \\ 4 - 2B &= +6A - 6 \end{aligned}\right\} \implies \left.\begin{aligned} -16 - 2B &= A + 4 \\ 4 - 2B &= 6A - 6 \end{aligned}\right\} \implies \text{SUBTRACT UPWARDS}$$

$$20 = 5A - 10$$

$$5A = 30$$

$$A = 6$$

C1, IYGB, PAPER Y

$$\& \quad -16 - 2B = A + 4$$
$$-16 - 2B = 6 + 4$$
$$-26 = 2B$$
$$B = -13$$

b)
$$u_{n+1} = \frac{6u_n + 2}{4 - 13u_n}$$

$$u_4 = \frac{6u_3 + 2}{4 - 13u_3} = \frac{6\left(-\frac{1}{3}\right) + 2}{4 - 13\left(-\frac{1}{3}\right)} = 0$$

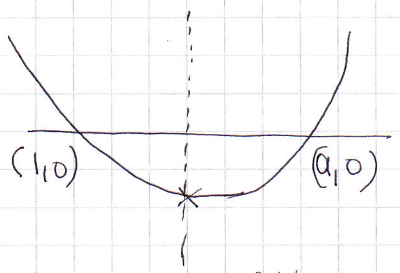
$$u_5 = \frac{6u_4 + 2}{4 - 13u_4} = \frac{6 \times 0 + 2}{4 - 13 \times 0} = \frac{1}{2}$$

THUS

$\frac{1}{2}$	-2	$-\frac{1}{3}$	0
u_1	u_2	u_3	u_4
u_5	u_6	u_7	u_8
$\vdots$	$\vdots$	$\vdots$	$\vdots$
			u_{32}
u_{33}	u_{34}	u_{35}	u_{36}
u_{37}			

$$\sum_{r=1}^{37} u_r = 9\left(\frac{1}{2} - 2 - \frac{1}{3} + 0\right) + \frac{1}{2}$$
$$= \frac{9}{2} - 18 - 3 + \frac{1}{2}$$
$$= 5 - 18 - 3$$
$$= -16$$

10.



$x = \frac{a+1}{2}$

BY SYMMETRY THE MIDPOINT OF (1,0) & (a,0)

$$y = \left(\frac{a+1}{2} - 1\right)\left(\frac{a+1}{2} - a\right)$$
$$y = \left(\frac{a+1}{2} - \frac{2}{2}\right)\left(\frac{a+1}{2} - \frac{2a}{2}\right)$$
$$y = \frac{a-1}{2} \times \frac{-a+1}{2}$$
$$y = \frac{a-1}{2} \times -\frac{a-1}{2}$$
$$y = -\frac{(a-1)^2}{4}$$
$$\therefore \left(\frac{a+1}{2}, -\frac{(a-1)^2}{4}\right)$$