C1, IYGB, PAPER V

1. a)

$3x - 2y + 6 = 0$

$3x + 6 = 2y$

$y = \frac{3}{2}x + 3$

$\therefore$ GRAD IS $\frac{3}{2}$

b) GRAD l_2 IS $-\frac{2}{3}$

USING $A(2,6)$

$y - y_0 = m(x - x_0)$

$y - 6 = -\frac{2}{3}(x - 2)$

$3y - 18 = -2x + 4$

$3y + 2x = 22$

c)

- $3y + 2x = 22$

  $0 + 2x = 22$

  $x = 11$ $\therefore D(11,0)$

- $y = \frac{3}{2}x + 3$ $\therefore B(0,3)$

USING $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

- $|AB| = \sqrt{(6-3)^2 + (2-0)^2}$ $A(2,6)$ $B(0,3)$

  $= \sqrt{9+4} = \sqrt{13}$

- $|AD| = \sqrt{(6-0)^2 + (2-11)^2}$ $A(2,6)$ $D(11,0)$

  $= \sqrt{36+81} = \sqrt{117}$

  $= 3\sqrt{13}$

AREA $= 3\sqrt{13} \times \sqrt{13} = 3 \times 13 = 39$

2.

$\frac{dy}{dx} = (kx - 3)\sqrt{x}$

$\left.\frac{dy}{dx}\right|_{x=4} = 34$

$34 = (k \times 4 - 3) \times \sqrt{4}$

$34 = (4k - 3) \times 2$

$17 = 4k - 3$

$20 = 4k$

$\boxed{k = 5}$

$y = \int (5x - 3)x^{\frac{1}{2}}\, dx$

$y = \int 5x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\, dx$

$y = \frac{5}{\frac{5}{2}}x^{\frac{5}{2}} - \frac{3}{\frac{3}{2}}x^{\frac{3}{2}} + C$

$\boxed{y = 2x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + C}$

$(4, 40)$

$40 = 2 \times 4^{\frac{5}{2}} - 2 \times 4^{\frac{3}{2}} + C$

$40 = 2 \times 32 - 2 \times 8 + C$

$40 = 64 - 16 + C$

$C = -8$

$y = 2x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 8$

3. a) I) $(2+\sqrt{3})(2\sqrt{3}-3) = 4\sqrt{3} - 6 + (2\times 3) - 3\sqrt{3}$

$= 4\sqrt{3} \cancel{- 6 + 6} - 3\sqrt{3}$

$= \sqrt{3}$

II) $\dfrac{\sqrt{6}+3\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \dfrac{(\sqrt{6}+3\sqrt{2})(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} = \dfrac{6-\sqrt{12}+3\sqrt{12}-(3\times 2)}{6\cancel{-\sqrt{12}+\sqrt{12}}-2}$

$= \dfrac{6+2\sqrt{12}-6}{4} = \dfrac{2\sqrt{12}}{4} = \dfrac{\sqrt{12}}{2} = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$

b) $8w^{\frac{1}{2}} - w^{-1} = 0$

$\Rightarrow 8w^{\frac{1}{2}} = \dfrac{1}{w}$

$\Rightarrow 8w^{\frac{1}{2}}w = 1$

$\Rightarrow 8w^{\frac{3}{2}} = 1$

$\Rightarrow w^{\frac{3}{2}} = \dfrac{1}{8}$

$\Rightarrow \left(w^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(\dfrac{1}{8}\right)^{\frac{2}{3}}$

$\Rightarrow w^{1} = \left(\sqrt[3]{\dfrac{1}{8}}\right)^{2}$

$\Rightarrow w = \left(\dfrac{1}{2}\right)^{2}$

$\Rightarrow w = \dfrac{1}{4}$

4. a) $f(x) = 5x^2 - 30x + 50$

$f(x) = 5\left[x^2 - 6x + 10\right]$

$f(x) = 5\left[(x-3)^2 - 3^2 + 10\right]$

$f(x) = 5\left[(x-3)^2 + 1\right]$

$f(x) = 5(x-3)^2 + 5$

It $a = 5$

$b = -3$

$c = 5$

b) MINIMUM VALUE IS 5

c) $A(5,6)$ $B(x, 2x+1)$

$|AB| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$|AB| = \sqrt{(x-5)^2 + (2x+1-6)^2}$

$|AB| = \sqrt{(x-5)^2 + (2x-5)^2}$

$|AB|^2 = (x-5)^2 + (2x-5)^2$

$|AB|^2 = x^2 - 10x + 25$
$\quad\quad 4x^2 - 20x + 25$

$|AB|^2 = 5x^2 - 30x + 50$ AS REQUIRED

d) $|AB|_{MIN} = \sqrt{5}$

e) IT OCCURS WHEN $x = 3$

$\therefore B(3,7)$

5. a)

$$y_{n+2} = y_{n+1} + 2y_n$$

$$y_1 = 1$$
$$y_2 = 5$$
$$y_3 = y_2 + 2y_1 = 5 + 2\times 1 = 7$$
$$y_4 = y_3 + 2y_2 = 7 + 2\times 5 = 17$$
$$y_5 = y_4 + 2y_3 = 17 + 2\times 7 = 31$$
$$y_6 = y_5 + 2y = 31 + 2\times 17 = 65$$

b)

$x_n = 2^n$	$\Rightarrow$	2	4	8	16	32	64
		2^1	2^2	2^3	2^4	2^5	2^6
		$\downarrow -1$	$\downarrow +1$	$\downarrow -1$	$\downarrow +1$	$\downarrow -1$	$\downarrow +1$
$y_n = ?$	$\Rightarrow$	1	5	7	17	31	65

$$\therefore\ y_n = 2^n + (-1)^n$$

6. a)

$$f(x) = \sqrt{8x^3 - 15}$$

$$f\left(\tfrac{1}{2}x\right) = \sqrt{8\left(\tfrac{1}{2}x\right)^3 - 15} = \sqrt{8\left(\tfrac{1}{8}x^3\right) - 15} = \sqrt{x^3 - 15}$$

$\therefore$ HORIZONTAL STRETCH BY SCALE FACTOR OF 2

b)

$$g(x) = f(x-1) + 15 \quad \leftarrow \quad \xrightarrow{1}\ \uparrow 15$$

$$g(3) = f(2) + 15$$

$$g(3) = \sqrt{8\times 2^3 - 15} + 15 = \sqrt{49} + 15 = 7 + 15 = 22$$

7. $\boxed{x^2 - 4ax + (2b+1) = 0}$

NO REAL ROOTS, $b^2 - 4ac < 0$

$$(-4a)^2 - 4 \times 1 \times (2b+1) < 0$$

$$16a^2 - 4(2b+1) < 0$$

$$4a^2 - (2b+1) < 0$$

$$-(2b+1) < -4a^2$$

$$2b+1 > 4a^2$$

$$2b > 4a^2 - 1$$

$$2b > (2a-1)(2a+1)$$

$$b > \frac{1}{2}(2a-1)(2a+1)$$

AS REQUIRED

8. $\boxed{y = \frac{2x^4 - x + 6}{6x} = \frac{2x^4}{6x} - \frac{x}{6x} + \frac{6}{6x} = \frac{1}{3}x^3 - \frac{1}{6} + x^{-1}}$

- $\frac{dy}{dx} = x^2 - x^{-2} = x^2 - \frac{1}{x^2}$

- $4y = 15x \Rightarrow y = \frac{15}{4}x$

- $\frac{dy}{dx} = \frac{15}{4}$

$$\Rightarrow x^2 - \frac{1}{x^2} = \frac{15}{4}$$

$$\Rightarrow x^4 - 1 = \frac{15}{4}x^2$$

$$\Rightarrow 4x^4 - 4 = 15x^2$$

$$\Rightarrow 4x^4 - 15x^2 - 4 = 0$$

$$\Rightarrow (4x^2+1)(x^2-4) = 0$$

$$\Rightarrow x^2 = \begin{cases} 4 \\ \cancel{-\frac{1}{4}} \end{cases}$$

$$\Rightarrow x = \begin{cases} 2 \\ -2 \end{cases} \qquad y = \begin{cases} \frac{2 \times 2^4 - 2 + 6}{6 \times 2} = 3 \\ \frac{2(-2)^4 - (-2) + 6}{6(-2)} = \frac{-10}{3} \end{cases}$$

THUS $(2,3)$ GRAD $\frac{15}{4}$ OR $\left(-2,-\frac{10}{3}\right)$ GRAD $\frac{15}{4}$

l_3: $y-3=\frac{15}{4}(x-2)$

$4y-12=15x-30$

$4y=15x-18$

l_2: $y+\frac{10}{3}=\frac{15}{4}(x+2)$

$12y+40=45x+90$

$12y=45x+50$

9.

u_1	u_2	u_3	u_4
2	$2b+3c$	$b-3c+1$	$4b+5c$

$+d$ $\quad$ $+d$ $\quad$ $+d$

$(2b+3c)-(2)=d \Rightarrow 2b+3c-2=d$

$(b-3c+1)-(2b+3c)=d \Rightarrow -b-6c+1=d$

$(4b+5c)-(b-3c+1)=d \Rightarrow 3b+8c-1=d$

$\Rightarrow$

$\left.\begin{array}{l}2b+3c-2=-b-6c+1\\ 3b+8c-1=-b-6c+1\end{array}\right\} \Rightarrow \left.\begin{array}{l}3b+9c-3=0\\ 4b+14c-2=0\end{array}\right\} \Rightarrow$

$\left.\begin{array}{l}b+3c=1\\ 2b+7c=1\end{array}\right\} \Rightarrow b=1-3c$

$\Rightarrow 2(1-3c)+7c=1$

$\Rightarrow 2-6c+7c=1$

$\boxed{c=-1}$ $\boxed{b=4}$

$\therefore$ u_1, u_2, u_3, u_4

$2, 5, 8, 11$

$\therefore$ $a=2$, $d=3$, $n=30$

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$S_{30}=\frac{30}{2}\left[2\times2+29\times3\right]$

$S_{30}=15\times91=910+455=1365$

AS REQUIRED