C1, IYGB, PAPER P

1.
$$3x - \frac{5}{x} = 2$$
$$\Rightarrow 3x^2 - 5 = 2x$$
$$\Rightarrow 3x^2 - 2x - 5 = 0$$
$$\Rightarrow (3x - 5)(x + 1) = 0$$
$$x = \begin{cases} -1 \\ \frac{5}{3} \end{cases}$$

2.
$$\sqrt{3}(x - \sqrt{3}) = x + \sqrt{3}$$
$$\sqrt{3}x - 3 = x + \sqrt{3}$$
$$\sqrt{3}x - x = 3 + \sqrt{3}$$
$$x(\sqrt{3} - 1) = 3 + \sqrt{3}$$
$$x = \frac{3 + \sqrt{3}}{\sqrt{3} - 1}$$
$$x = \frac{(3 + \sqrt{3})(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$
$$x = \frac{3\sqrt{3} + 3 + 3 + \sqrt{3}}{3 + \cancel{\sqrt{3}} - \cancel{\sqrt{3}} - 1}$$
$$x = \frac{4\sqrt{3} + 6}{2}$$
$$x = 3 + 2\sqrt{3}$$

3. a) $u_5 = 38$ $\qquad$ $u_{20} = 158$

$$\boxed{u_n = a + (n-1)d}$$

$38 = a + 4d$ $\qquad$ $158 = a + 19d$

$a = 38 - 4d$ $\qquad$ $a = 158 - 19d$

$$38 - 4d = 158 - 19d$$
$$15d = 120$$
$$d = 8$$

$$a = 38 - 4 \times 8 = 6$$
$$a = 6$$

b)
$$S_n = \frac{n}{2}[a + l]$$
$$S_{20} = \frac{20}{2}[6 + 158]$$
$$S_{20} = 10 \times 164$$
$$S_{20} = 1640$$

or
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{20} = \frac{20}{2}[2 \times 6 + 19 \times 8]$$
$$S_{20} = 10(12 + 152)$$
$$S_{20} = 1640$$

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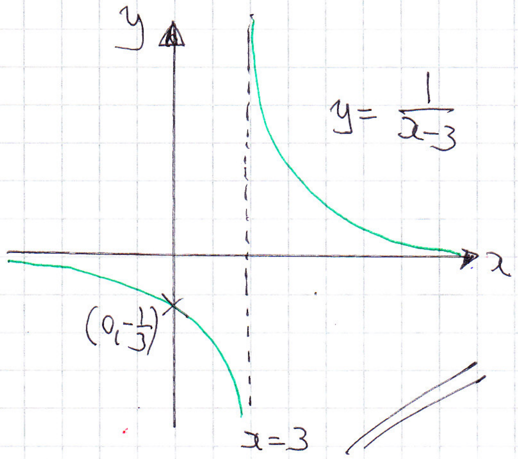
4.
$$xy = 3 \quad (\times 3)$$
$$3x + y = 10 \quad (\times y)$$
$$\Rightarrow \begin{matrix} 3xy = 9 \\ 3xy + y^2 = 10y \end{matrix} \Big\} \Rightarrow 9 + y^2 = 10y$$
$$\Rightarrow y^2 - 10y + 9 = 0$$
$$\Rightarrow (y-9)(y-1) = 0$$
$$y = \begin{cases} 1 \\ 9 \end{cases}$$
$$\boxed{x = \frac{3}{y}}$$
$$x = \begin{cases} \frac{3}{1} = 3 \\ \frac{3}{9} = \frac{1}{3} \end{cases}$$
$(3, 1)$ OR $\left(\frac{1}{3}, 9\right)$

5.

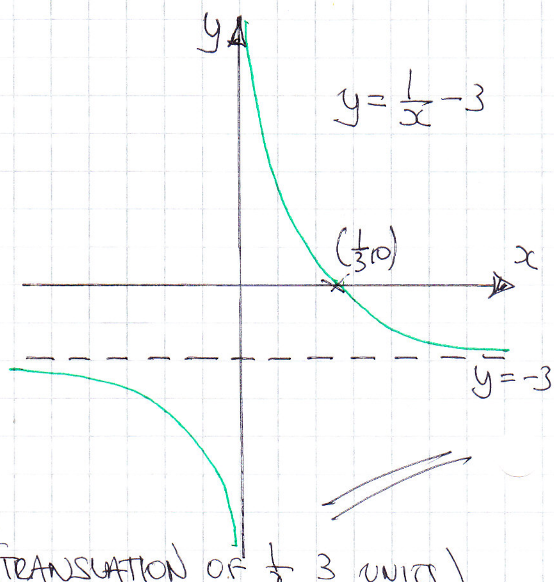
USING $P(2,k)$ WITH $y = 2x + 7$
$$k = 2 \times 2 + 7$$
$$k = 11$$

USING $P(2,11)$ WITH $y = 3x + C$
$$11 = 3 \times 2 + C$$
$$11 = 6 + C$$
$$C = 5$$

6.



(TRANSLATION OF $y = \frac{1}{x}$, 3 UNITS TO THE "RIGHT")



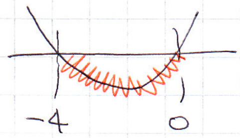
(TRANSLATION OF $\frac{1}{x}$ 3 UNITS "DOWNWARDS")

7.

$$m(1-x)-x^2=0$$
$$m-mx-x^2=0$$
$$0=x^2+mx-m$$

NO REAL ROOTS $\Rightarrow b^2-4ac<0$
$\Rightarrow m^2-4\times1\times(-m)<0$
$\Rightarrow m^2+4m<0$
$\Rightarrow m(m+4)<0$

C.V = $\left\langle \begin{matrix} 0 \\ -4 \end{matrix}\right.$



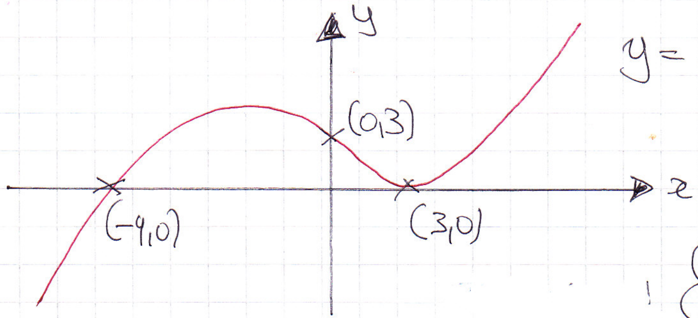
$-4<m<0$

8. a)

$$f(x)=(x+3)(x-1)^2$$
$$f(x)=(x+3)(x^2-2x+1)$$
$$f(x)=x^3-2x^2+x$$
$$\qquad\quad 3x^2-6x+3$$
$$\overline{x^3+x^2-5x+3}$$

$a=1$
$b=-5$
$c=3$

b)



$y=f(\tfrac{1}{3}x)$

HORIZONTAL STRETCH BY SCALE FACTOR 3

c) $\begin{pmatrix}-1\\0\end{pmatrix}$ IS A TRANSLATION ONE UNIT TO THE "LEFT"

$$g(x)=f(x+1)=\left[(x+1)+3\right]\left[(x+1)-1\right]^2$$
$$=(x+4)x^2$$
$$=x^3+4x^2$$

AS REQUIRED

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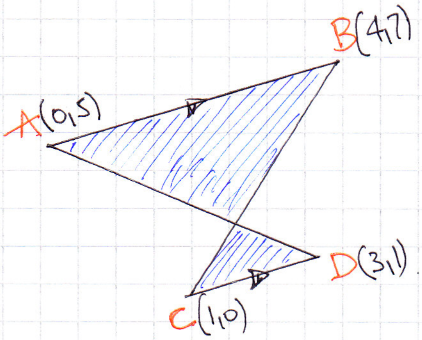
9. a) GRAD AB $= \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{7-5}{4-0} = \dfrac{2}{4} = \dfrac{1}{2}$

EQUATION OF l_1 USING $(0,5)$ IS $y = \frac{1}{2}x + 5$

b) GRAD CD $= \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{1-0}{3-1} = \dfrac{1}{2}$

AS THE GRADIENTS ARE EQUAL l_1 IS PARALLEL TO l_2

c)



$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

- $|AB| = \sqrt{(7-5)^2 + (4-0)^2}$

  $|AB| = \sqrt{4+16} = \sqrt{20}$

- $|CD| = \sqrt{(1-0)^2 + (3-1)^2} = \sqrt{5}$

$\dfrac{|AB|}{|CD|} = \dfrac{\sqrt{20}}{\sqrt{5}} = \dfrac{2\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}}$

$\therefore$ LENGTH SCALE FACTOR IS 2

AREA OF ABE : AREA OF ECD

4 : 1

10. a) $\dfrac{dy}{dx} = \dfrac{x^{\frac{5}{2}}+24}{x^2} = \dfrac{x^{\frac{5}{2}}}{x^2} + \dfrac{24}{x^2} = x^{\frac{1}{2}} + 24x^{-2}$

$\left.\dfrac{dy}{dx}\right|_{x=4} = \dfrac{4^{\frac{5}{2}}+24}{4^2} = \dfrac{32+24}{16} = \dfrac{56}{16} = \dfrac{7}{2}$

EQUATION OF TANGENT: $y - y_0 = m(x-x_0)$

$y - \frac{1}{3} = \frac{7}{2}(x-4)$

OR $\times 6$

$6y - 2 = 21(x-4)$

$6y - 2 = 21x - 84$

$6y = 21x - 82$

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b)

$$\frac{dy}{dx} = x^{\frac{1}{2}} + 24x^{-2}$$

$$y = \int x^{\frac{1}{2}} + 24x^{-2}\,dx = \frac{2}{3}x^{\frac{3}{2}} - 24x^{-1} + C$$

$$\therefore y = \frac{2}{3}x^{\frac{3}{2}} - \frac{24}{x} + C$$

$$\left(4, \frac{1}{3}\right) \Longrightarrow \frac{1}{3} = \frac{2}{3} \times 4^{\frac{3}{2}} - \frac{24}{4} + C$$

$$\frac{1}{3} = \frac{16}{3} - 6 + C$$

$$6 + \frac{1}{3} - \frac{16}{3} + C$$

$$C = 1$$

$$\therefore y = \frac{2}{3}x^{\frac{3}{2}} - \frac{24}{x} + 1$$

11.

$$y = 2x^3 - 4x^2 + 2x - 1$$

$$\frac{dy}{dx} = 6x^2 - 8x + 2$$

$$x + 2y + 1 = 0$$

$$2y = -x - 1$$

$$y = -\frac{1}{2}x - \frac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{x=p} = -\frac{1}{2}$$

$$6p^2 - 8p + 2 = -\frac{1}{2}$$

$$6p^2 - 8p + \frac{5}{2} = 0$$

$$12p^2 - 16p + 5 = 0$$

$$(6p - 5)(2p - 1) = 0$$

$$p = \begin{cases} \frac{1}{2} \\ \frac{5}{6} \end{cases} \xRightarrow{y = -\frac{1}{2}x - \frac{1}{2}} q = \begin{cases} -\frac{1}{2}\left(\frac{1}{2}\right) - \frac{1}{2} = -\frac{3}{4} \\ -\frac{1}{2}\left(\frac{5}{6}\right) - \frac{1}{2} = -\frac{5}{12} - \frac{1}{2} \\ -\frac{5}{12} - \frac{6}{12} = -\frac{11}{12} \end{cases}$$

EITHER $\left(\frac{1}{2}, -\frac{3}{4}\right)$ OR $\left(\frac{5}{6}, -\frac{11}{12}\right)$

CHECK $\left(\frac{1}{2}, -\frac{3}{4}\right)$ WITH CUBIC

$$y = 2x^3 - 4x^2 + 2x - 1$$
$$y = 2\left(\tfrac{1}{2}\right)^3 - 4\left(\tfrac{1}{2}\right)^2 + 2\left(\tfrac{1}{2}\right) - 1$$
$$y = \frac{1}{4} - 1 \cancel{+1} \cancel{-1}$$
$$y = -\frac{3}{4}$$

$$\therefore \quad p = \frac{1}{2}$$
$$q = -\frac{3}{4}$$