C1, IYGB, PAPER M

1. $\int y\,dx = \int 3x^2 - 6x^{\frac{1}{2}} - x^{-2} + 4\,dx$

$$= x^3 - \left(\frac{\frac{6}{1}}{\frac{3}{2}}\right)x^{\frac{3}{2}} + x^{-1} + 4x + C = x^3 - 4x^{\frac{3}{2}} + x^{-1} + 4x + C$$

2. a) $(\sqrt{7}+2)(1+\sqrt{7}) = \sqrt{7} + 7 + 2 + 2\sqrt{7} = 9 + 3\sqrt{7}$

b) $\frac{\sqrt{50}+\sqrt{18}}{\sqrt{8}} = \frac{\sqrt{25}\sqrt{2}+\sqrt{9}\sqrt{2}}{\sqrt{4}\sqrt{2}} = \frac{5\sqrt{2}+3\sqrt{2}}{2\sqrt{2}} = \frac{8\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}} = 4$

3. $\sum_{r=1}^{20}(13r+4) = \underbrace{17 + 30 + 43 + \dots + 264}_{\text{This is an A.P. with}}$

$a = 17$
$d = 13$
$l = 264$
$n = 20$

$S_n = \frac{n}{2}[a+l]$

$S_{20} = \frac{20}{2}[17+264]$

$S_{20} = 10 \times 281$

$S_{20} = 2810$

OR

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{20} = \frac{20}{2}[2\times 17 + 19\times 13]$

$S_{20} = 10[34 + 247]$

$S_{20} = 2810$

4. $f'(x) = 6x^2 - 4x$

$f(x) = \int 6x^2 - 4x\,dx$

$\boxed{f(x) = 2x^3 - 2x^2 + C}$

when $x=1$ $y=3$

$3 = 2\times 1^3 - 2\times 1^2 + C$

$3 = 2 - 2 + C$

$C = 3$

Thus $f(x) = 2x^3 - 2x^2 + 3$

5. a) $\left.\begin{array}{l} x^2 = 4y + 12 \\ 2y - 3x = 2 \end{array}\right\} \Rightarrow \begin{array}{l} 2y = 2 + 3x \\ 4y = 4 + 6x \end{array}$

SUBSTITUTE INTO THE OTHER

$x^2 = (4 + 6x) + 12$

$x^2 = 6x + 16$

$x^2 - 6x - 16 = 0$

$(x+2)(x-8) = 0$

$x = \begin{cases} -2 \\ 8 \end{cases}$

- $2y = 2 + 3x$
  $2y = 2 + 3(-2)$
  $2y = 2 - 6$
  $2y = -4$
  $y = -2$
- $2y = 2 + 3x$
  $2y = 2 + 3 \times 8$
  $2y = 26$
  $y = 13$

$\therefore P(-2,-2) \quad Q(8,13)$

b) $A(0,-3)$ ← $\boxed{\begin{array}{l} x = 0 \\ 0 = 4y + 12 \\ y = -3 \end{array}}$

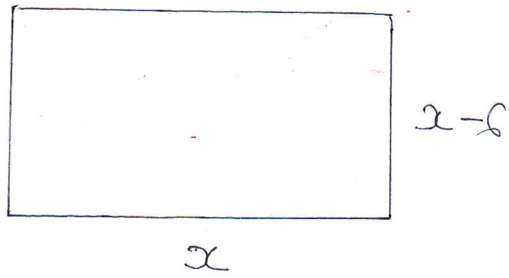
$P(-2,-2)$

$Q(8,13)$

- GRADIENT AP $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - (-3)}{-2 - 0} = \dfrac{1}{-2} = -\dfrac{1}{2}$
- GRADIENT AQ $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{13 - (-3)}{8 - 0} = \dfrac{16}{8} = 2$
- AS THE GRADIENTS ARE NEGATIVE RECIPROCALS OF EACH OTHER $AP \perp AQ$
  $\therefore \hat{PAQ} = 90°$

(P.T.O)

6.



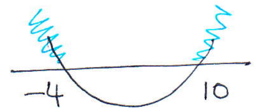
LET x = LENGTH!!

$x(x-6) > 40$

$x^2 - 6x - 40 > 0$

$(x+4)(x-10) > 0$

C.V = -4, 10



$x < -4$ OR $x > 10$

BUT x IS A LENGTH

$\therefore x > 10$

7. a) $a = 300$, $d = 5$

$u_n = a + (n-1)d$

$u_{12} = 300 + 11 \times 5$

$u_{12} = 355$

b) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{48} = \frac{48}{2}\left[2 \times 300 + 47 \times 5\right]$

$S_{48} = 24\left[600 + 235\right]$

$S_{48} = 24 \times 835$

$S_{48} = 20040$

$$\begin{array}{r} 835 \\ 24 \\ \hline 3340 \\ 1770 \\ \hline 20040 \end{array}$$

c) $a = ?$, $d = 15$, $S_{48} = 20040$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$20040 = \frac{48}{2}\left[2a + 47 \times 15\right]$

$20040 = 24\left[2a + 705\right]$

$835 = 2a + 705$

$130 = 2a$

$a = 65$

$$\begin{array}{r} 470 \\ 235 \\ \hline 705 \end{array}$$

$$\begin{array}{r|l} 20040 & 24 \\ 84 & 835 \\ 120 & \end{array}$$

8. a) Touches $\Rightarrow b^2-4ac=0$

$\Rightarrow (2m)^2-4\times1\times(3m+4)=0$

$\Rightarrow 4m^2-4(3m+4)=0$

$\Rightarrow 4m^2-12m-16=0$

$\Rightarrow m^2-3m-4=0$

$\Rightarrow (m+1)(m-4)=0$

$\Rightarrow m = -1,\ 4$

b) • If $m=-1$

$y=x^2-2x+1$

$y=(x-1)^2$

$\therefore (1,0)$

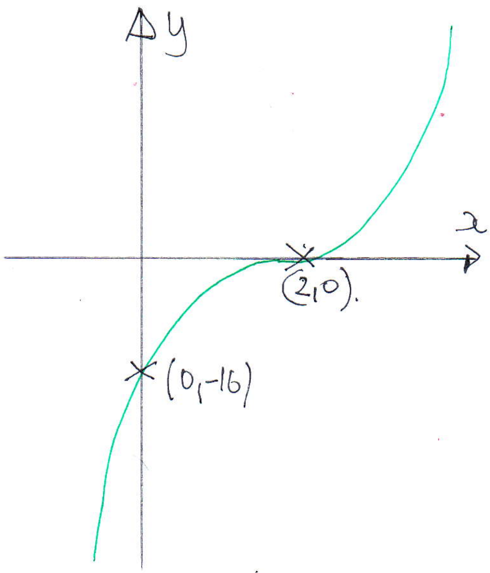
• If $m=4$

$y=x^2+8x+16$

$y=(x+4)^2$

$\therefore (-4,0)$

9. a)



b) $f(x)=2(x-2)^3$

$f(x)=2(x-2)(x-2)^2$

$f(x)=(2x-4)(x^2-4x+4)$

$f(x)=2x^3-8x^2+8x$
$\qquad -4x^2+16x-16$

$f(x)=2x^3-12x^2+24x-16$

$\therefore f'(x)=6x^2-24x+24$

c) WHEN $x=3$

$$f'(3) = 6\times3^2 - 24\times3 + 24$$
$$= 54 - 72 + 24$$
$$= 6$$

USING $(3,2)$

$$y - y_0 = m(x - x_0)$$
$$y - 2 = 6(x-3)$$
$$y = 6x - 18 + 2$$
$$y = 6x - 16$$

d) PARALLEL $\Rightarrow$ SAME GRADIENT $\Rightarrow f'(x) = 6$

$$\Rightarrow 6x^2 - 24x + 24 = 6$$
$$\Rightarrow 6x^2 - 24x + 18 = 0$$
$$\Rightarrow x^2 - 4x + 3 = 0$$
$$\Rightarrow (x-3)(x-1) = 0$$

$x = 3 \leftarrow P$, $x = 1 \leftarrow Q$

$$y = 2(1-2)^3 = -2$$

$\therefore Q(1,-2)$

THUS

$$y - y_0 = m(x - x_0)$$
$$y + 2 = 6(x-1)$$
$$y + 2 = 6x - 6$$
$$y = 6x - 8$$

AS REQUIRED

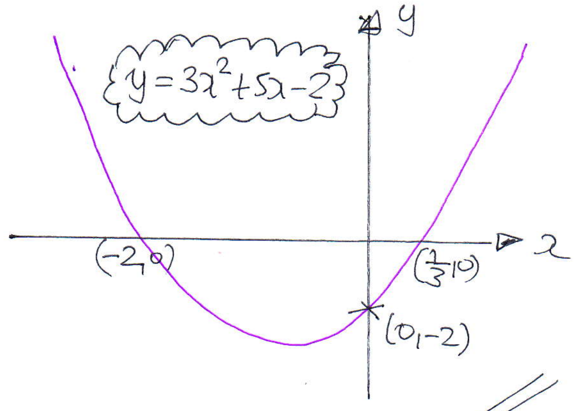
C1, IYGB, PAPER M

10. (a) $f(x)=0$

$$3x^2+5x-2=0$$

$$(3x-1)(x+2)=0$$

$$x = \begin{cases} 1/3 \\ -2 \end{cases}$$



(c) $f(\frac{1}{3}x)$ IS A HORIZONTAL STRETCH BY S.F. 3

THUS $(-2,0) \longmapsto (-6,0)$

$(\frac{1}{3},0) \longmapsto (1,0)$

$(0,-2) \longmapsto (0,-2)$

d) TRANSLATION 1 UNIT "LEFT" $\Longrightarrow f(x+1)$

$$f(x+1) = 3(x+1)^2+5(x+1)-2$$

$$f(x+1) = 3(x^2+2x+1)+5x+5-2$$

$$f(x+1) = 3x^2+6x+3+5x+5-2$$

$$\therefore y = 3x^2+11x+6$$

ALTERNATIVE

$f(x)$ CROSSES AT $(-2,0)$ & $(\frac{1}{3},0)$

$f(x+1)$ MUST CROSS AT $(-3,0)$ & $(-\frac{2}{3},0)$

$$\therefore y=(x+3)(3x+2)$$

$$y = 3x^2+11x+6$$

C1, IYGB, PAPER M

11. a) $2^{x+1} + 2^{3-x} = 17$

$2^x \times 2^1 + 2^3 \times 2^{-x} = 17$

$2(2^x) + 8\left(\frac{1}{2^x}\right) = 17$

LET $y = 2^x$

$2y + \frac{8}{y} = 17$

$2y^2 + 8 = 17y$

$2y^2 - 17y + 8 = 0$

b) $2y^2 - 17y + 8 = 0$

$(2y-1)(y-8) = 0$

$y = \frac{1}{2}$ or 8

$2^x = \frac{1}{2}$ or 8

$x = -1$ or 3