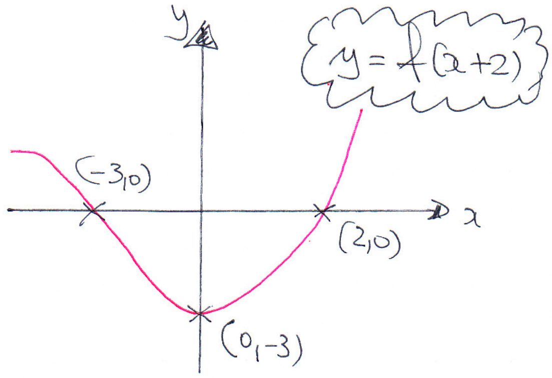
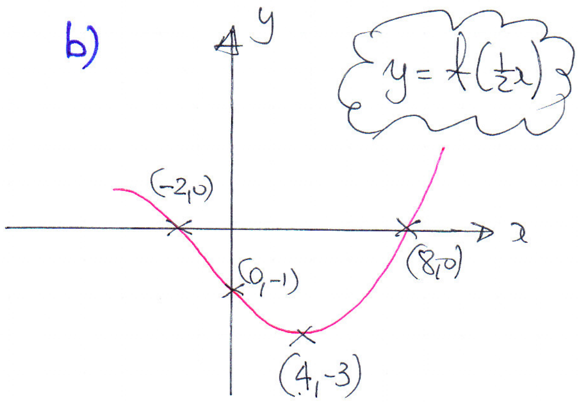
C1, IYGB, PAPER L

1. a)



TRANSLATION, 2 UNITS TO THE "LEFT"

b)



HORIZONTAL STRETCH BY FACTOR OF 2

2.
$$\frac{1+\sqrt{7}}{3-\sqrt{7}} - \frac{8-\sqrt{7}}{\sqrt{7}-2} = \frac{(1+\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} - \frac{(8-\sqrt{7})(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$$

$$= \frac{3+\sqrt{7}+3\sqrt{7}+7}{9+\cancel{3\sqrt{7}}-\cancel{3\sqrt{7}}-7} - \frac{8\sqrt{7}+16-7-2\sqrt{7}}{7+\cancel{2\sqrt{7}}-\cancel{2\sqrt{7}}-4}$$

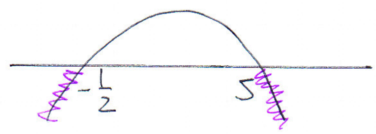
$$= \frac{10+4\sqrt{7}}{2} - \frac{6\sqrt{7}+9}{3}$$

$$= (5+2\sqrt{7}) - (2\sqrt{7}+3) = 2$$

3. a)
$$4(2x+3)+x > 47-5x$$
$$8x+12+x > 47-5x$$
$$14x > 35$$
$$x > \frac{35}{14}$$
$$x > \frac{5}{2}$$

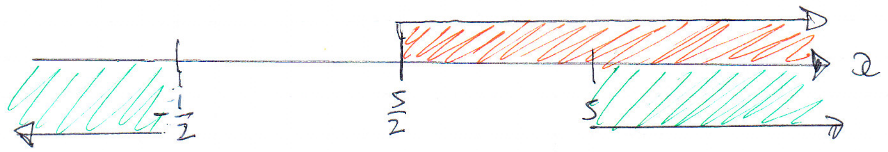
b)
$$(5-x)(2x+1) \le 0$$

$$\text{C.V} = \begin{cases} 5 \\ -\frac{1}{2} \end{cases}$$



$x \le -\frac{1}{2}$ OR $x \ge 5$

c)



$x \ge 5$

4.

$$\sum_{r=1}^{50}(180-7r) = \underbrace{173+166+159+\dots+(-170)}_{\text{THIS IS AN A.P}}$$

WITH $a = 173$, $d = -7$, $l = -170$, $n = 50$

$S_n = \frac{n}{2}\left[a + l\right]$

$S_{50} = \frac{50}{2}\left[173 - 170\right]$

$S_{50} = 25 \times 3$

$S_{50} = 75$

OR

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{50} = \frac{50}{2}\left[2\times173 + 49(-7)\right]$

$S_{50} = 25\left[346 - 280 - 63\right]$

$S_{50} = 25\left[346 - 343\right]$

$S_{50} = 75$

5. (a)

$\left.\begin{array}{l} y = x(6-x) \\ 2y = 7x+10 \end{array}\right\} \Rightarrow 2\left[x(6-x)\right] = 7x+10$

$2x(6-x) = 7x+10$

$12x - 2x^2 = 7x + 10$

$0 = 2x^2 - 5x + 10$

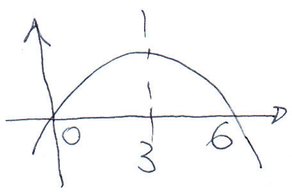
$b^2 - 4ac = (-5)^2 - 4\times2\times10$

$= 25 - 80$

$= -55 < 0$

NO ROOTS $\Rightarrow$ NO INTERSECTIONS

(b) COMPLETE THE SQUARE AFTER EXPANDING OR USE SYMMETRY

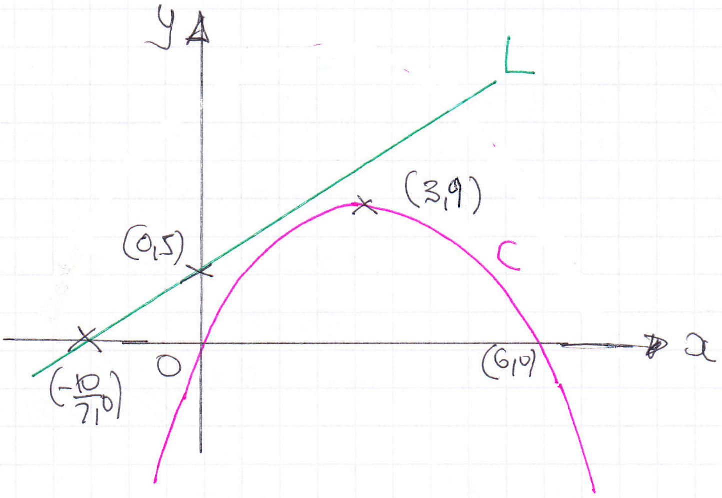


MAX OCCURS WHEN $x = 3$

$y = x(6-x) = 3(6-3) = 3\times3 = 9$

$\therefore (3,9)$

c)



$2y = 7x + 10$

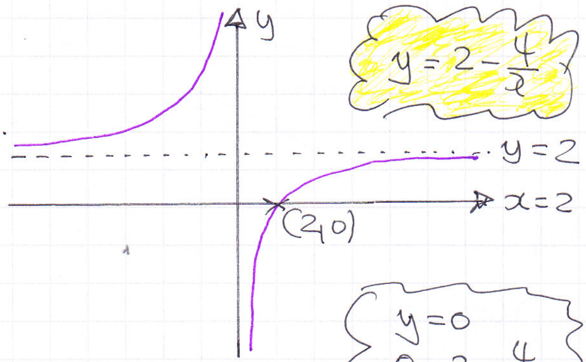
$y = \frac{7}{2}x + 5$

$x = 0,\ y = 5 \quad (0,5)$

$y = 0,\ 7x + 10 = 0$

$x = -\frac{10}{7} \quad \left(-\frac{10}{7},0\right)$

6. a) TRANSLATION "UP" BY 2 UNITS



$y = 2 - \frac{4}{x}$

$y = 0$

$0 = 2 - \frac{4}{x}$

$\frac{4}{x} = 2$

$4 = 2x$

$x = 2$

b) ASYMPTOTES

- $y = 2$

&

- y AXIS OR $x = 0$

7.

$$\frac{dy}{dx} = 8\sqrt[3]{x} - 10$$

$$y = \int 8x^{\frac{1}{3}} - 10 \, dx$$

$$y = \frac{8}{\frac{4}{3}}x^{\frac{4}{3}} - 10x + C$$

$$y = 6x^{\frac{4}{3}} - 10x + C$$

NOW $x = 8,\ y = 18$

$$18 = 6 \times 8^{\frac{4}{3}} - 10 \times 8 + C$$

$$18 = 6 \times 16 - 80 + C$$

$$18 = 96 - 80 + C$$

$$18 = 16 + C$$

$$C = 2$$

$$\therefore y = 6x^{\frac{4}{3}} - 10x + 2$$

8. a) If ARITHMETIC $\left.\begin{array}{l} u_2 - u_1 = d \\ u_3 - u_2 = d \end{array}\right\} \Rightarrow u_2 - u_1 = u_3 - u_2$

$\Rightarrow (2p-5) - (-p) = 3p - 2 - (2p-5)$

$\Rightarrow 3p - 5 = p + 3$

$\Rightarrow 2p = 8$

$\Rightarrow p = 4$ AS REQUIRED

b) Thus

$u_1 = -4$
$u_2 = 3$ (+7)
$u_3 = 10$ (+7)

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{20} = \frac{20}{2}\left[2(-4) + 19 \times 7\right]$

$S_{20} = 10\left[-8 + 70 + 63\right]$

$S_{20} = 1250$

c) $u_n = a + (n-1)d$

$1000 = -4 + (k-1) \times 7$

$1004 = 7k - 7$

$1011 = 7k$

$k = \frac{1011}{7} = \frac{700 + 280 + 28 + 3}{7} = 100 + 40 + 4 + \frac{3}{7} = 144\frac{3}{7}$

$\therefore k = 145$

9. a) GRADIENT OF AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{12-0} = \frac{6}{12} = \frac{1}{2}$

LINE PASSES THROUGH $(0,3)$

$\therefore y = \frac{1}{2}x + 3$

OR

$2y = x + 6$

b) GRAD OF l_2 IS -2 (PERPENDICULAR)

$$y - y_0 = m(x - x_0)$$
$$y - 1 = -2(x - 11)$$
$$y - 1 = -2x + 22$$
$$l_2: \boxed{y = -2x + 23}$$
$$l_1: \boxed{2y = x + 6}$$

$$\Rightarrow 2(-2x+23) = x+6$$
$$-4x + 46 = x + 6$$
$$40 = 5x$$
$$\boxed{x = 8}$$

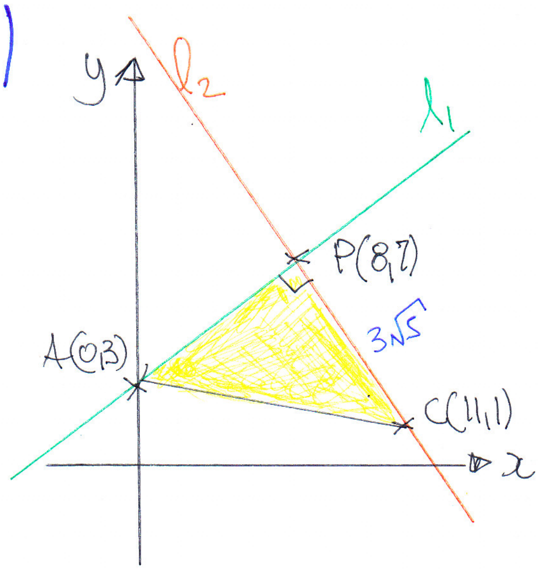
& $y = -2(8) + 23$
$$\boxed{y = 7}$$

$\therefore$ $P(8,7)$

c) $C(11,1)$ $P(8,7)$ $\Rightarrow$ $d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$$d = \sqrt{(7-1)^2 + (8-11)^2}$$
$$d = \sqrt{36 + 9}$$
$$d = \sqrt{45} \quad \text{OR} \quad 3\sqrt{5}$$

d)



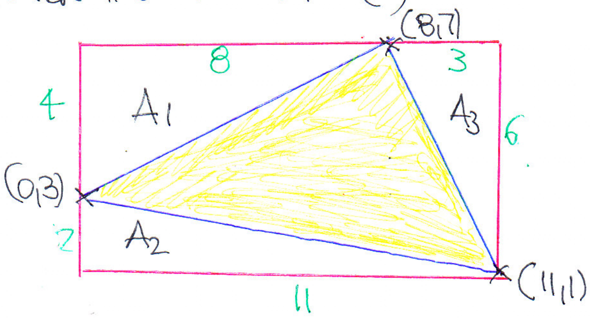
$$|AP| = \sqrt{(7-3)^2 + (8-0)^2}$$
$$|AP| = \sqrt{16 + 64}$$
$$|AP| = \sqrt{80}$$
$$|AP| = 4\sqrt{5}$$

THUS

$$\text{AREA} = \frac{1}{2}|AP||PC|$$
$$= \frac{1}{2} \times 4\sqrt{5} \times 3\sqrt{5}$$
$$= 2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30$$

C1, IYGB, PAPER L

ALTERNATIVE FOR PART (C)



- RECTANGLE $= 6\times 11 = 66$

$A_1 = \frac{1}{2}\times 4\times 8 = 16$

$A_2 = \frac{1}{2}\times 2\times 11 = 11$

$A_3 = \frac{1}{2}\times 3\times 6 = 9$

36

$\therefore$ TRIANGLE AREA $= 66-36 = 30$

10. a) $\left.\begin{array}{l} y = 2x^2-6x+5 \\ 2y+x=4 \end{array}\right\} \Rightarrow 2(2x^2-6x+5)+x=4$

$\Rightarrow 4x^2-12x+10+x=4$

$\Rightarrow 4x^2-11x+6=0$

$\Rightarrow (4x-3)(x-2)=0$

$x = \left\langle\begin{array}{l} 2 \\ \frac{3}{4} \end{array}\right.$

NOW $2y+2=4$, $2y=2$, $y=1$

$2y+\frac{3}{4}=4$, $8y+3=16$, $8y=13$, $y=\frac{13}{8}$

$\therefore (2,1)$ & $\left(\frac{3}{4}, \frac{13}{8}\right)$

b) • $2y+x=4$

$2y=-x+4$

$y = -\frac{1}{2}x+2$

• $y=2x^2-6x+5$

$\frac{dy}{dx} = 4x-6$

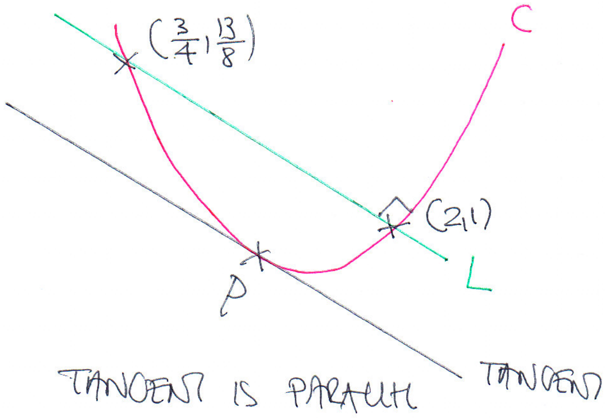
$\left.\frac{dy}{dx}\right|_{x=2} = 4\times 2-6 = 2$

GRAD OF TANGENT AT (2,1) IS 2

GRAD OF NORMAL AT (2,1) IS $-\frac{1}{2}$

C1, IYGB, PAPER L

AS L CROSSES C AT (2,1), L IS A NORMAL TO C
(SEE BELOW)



c) TANGENT IS PARALLEL TO THE LINE L

$\therefore$ TANGENT GRADIENT IS $-\frac{1}{2}$

$$\frac{dy}{dx} = 4x - 6 \quad \longleftarrow \text{ THIS GIVES THE GRADIENT OF TANGENT}$$

$$-\frac{1}{2} = 4x - 6$$

$$-1 = 8x - 12$$

$$11 = 8x$$

$$x = \frac{11}{8}$$